Target Scattering Predictions Using MAVART Finite Element Code

Christopher Purcell, Richard Fleming and John Fawcett
Defence Research and Development Canada - Atlantic
9 Grove Street
Dartmouth Nova Scotia, B2Y 3Z7
CANADA

chris.purcell@drdc-rddc.gc.ca / richard.fleming@drdc-rddc.gc.ca / john.fawcett@drdc-rddc.gc.ca

ABSTRACT

Defence Research and Development Canada – Atlantic has developed three versions of the Model to Analyze the Vibrations and Acoustic Radiation of Transducers (MAVART, MAVART3D and MAVARTMAG). These codes were applied to a model of a radiating cylinder and their predictions of acoustic power were in close agreement. MAVART3D was tested by comparison with the analytical solution to scattering from a hard sphere and was used to model scattering from a cylinder using the Double Asymptotic Approximation. The results were compared with predictions from the FE-VSC scattering code, developed at the NATO Undersea Research Centre. These two codes show good overall agreement with discrepancies likely due to different shell elements.

1.0 INTRODUCTION

The **M**odel to **A**nalyze the **V**ibrations and **A**coustic **R**adiation of **T**ransducers (MAVART) is a family of coupled physics finite element codes that have been under development and in use at DRDC Atlantic for more than 25 years. There are 3 members of the family: MAVART, MAVART3D and MAVARTMAG.

MAVART was developed during 1976-1996 to model axisymmetric piezoelectric transducers. It solves a linear problem in the frequency domain of the coupled interaction between electric potential applied to piezoelectric solids, with transversely isotropic material properties, Rayleigh damping, and solid to fluid interaction. Acoustic radiation is modelled by enclosing the transducer in a sphere of fluid finite elements, with attached "infinite elements" that match the pressures to a Legendre polynomial series solution for the outgoing waves. MAVART has been extensively tested and used to make design improvements to the multi-mode pipe projector, ring shell, resonant pipe, free-flooded ring, barrel stave and flared waveguide projectors and a number of hydrophones. There are 17 element types in the most recent version of the code (Version 14). MAVART can estimate transducer performance measures including transmitting voltage response (TVR), efficiency, directivity index and electrical admittance [1].

MAVART3D was developed during 1991-2001 to model 3D piezoelectric transducers, but it can also handle 2D, axisymmetric and n-fold symmetries [2]. Acoustic radiation is handled by the Double Asymptotic Approximation (DAA) or Boundary Element Method (BEM). There are 119 element types in the most recent version of the code (Version 4.3) including linear, quadratic and shell elements. Although it is a frequency domain, linear code, it has a limited nonlinear capability (prestress and large deformation). It can perform 3D or n-fold scattering analyses with DAA elements by adding the incident wave load to the load vector.

MAVARTMAG was developed during 2001-2003 to model electrodynamic transducers [3]. There are 102 element types in the most recent version of the code (Version 2.2) to deal with solids, fluids, fluid-solid interaction, acoustic radiation (using DAA), Ohmic conductors, permanent magnets, coils, and eddy currents. It will be used for designing loudspeakers, moving coil projectors, sensors, magnets, motors and actuators. Although it is a frequency domain, linear code, it has a nonlinear capability to establish the operating point of magnetic circuits.

Models for all the MAVART codes are built with a Mathematica package called ModelMaker [4]. This package can build parametric models that contain symbolic expressions as well as numeric data. Parametric models can be optimized using Mathematica's built-in or add-on optimization packages [5]. This capability played a key role in the development of the wide-band multi-mode pipe projector (MMPP) [6].

This paper will discuss the modelling of two simple scatterers: the rigid sphere and the hemispherical end-capped cylinder. We will verify that all 3 MAVART codes agree on a simple radiation problem, and we will compare results from MAVART3D with results from the FE-VSC scattering code [7], developed at the NATO Undersea Research Centre, and with analytic solutions. Our aim is to demonstrate that in addition to modelling transduction, MAVART3D can be used for modelling scattering.

All computations were performed with a 2.8 GHz Xeon processor, 400 MHz front side bus, and 4 GB RAM.

2.0 RIGID SPHERE SCATTERING

The classic problem of plane wave scattering from a hard sphere was used to test the DAA scattering algorithm in MAVART3D. The series solution for the scattered pressure [8] was coded in Mathematica (see Figure 1) as a truncated sum of spherical Hankel and Bessel functions using Mathematica's variable precision arithmetic to control round-off error.

```
H[n_, z_] := BesselJ[n, z] + i BesselY[n, z]; (* hankel function *)

h[n_, z_] := Sqrt[π/(2 z)] H[n + 1/2, z]; (* spherical hankel function *)

j[n_, z_] := Sqrt[π/(2 z)] BesselJ[n + 1/2, z]; (* spherical bessel function *)

Options[Ps] = {Radius → 1, Precision → Infinity};
Ps::usage = "Ps[r,θ,ka,m,opts] returns the scattered pressure from a hard sphere as a function
    of ka, at radius r and polar angle θ resulting from a plane wave of amplitude 1 Pa incident
    along the z axis. The pressure is calculated from the sum of the first m terms in the
    series solution. The radius of the sphere is set by the option Radius. The number of
    significant figures is set by the option Precision.";

Ps[r_, θ_, ka_, m_, opts___] := Module[{k, a, pa, pθ, pr, pka, Precisionval},
    {Precisionval, a} = {Precision, Radius} /. Flatten[{opts}] /. Options[Ps];
    pa = SetPrecision[a, Precisionval]; pr = SetPrecision[r, Precisionval];
    pθ = SetPrecision[θ, Precisionval]; pka = SetPrecision[ka, Precisionval];
    k = pka/pa; (* quotient of high precision numbers will also be high precision *)
    -N[Sum[((2 n + 1) i^n LegendreP[n, Cos[pθ]] (∂_x j[n, x] /. x → pka) h[n, k * pr]) / (∂_x h[n, x] /. x → pka),
        {n, 0, m}], Precisionval]];
```

Figure 1. Mathematica code for series solution of hard sphere scattering.

2.1 Rigid Sphere Model

A one-meter radius sphere model was constructed with ModelMaker using solid linear quadrilateral elements, fluid to solid elements, near-field fluid elements, fluid-to fluid dampers and infinite fluid elements. The problem is axisymmetric, but since MAVART3D does not have axisymmetric scattering implemented even though it does have axisymmetric radiation, we instead used its n-fold symmetry scattering capability to reduce the node count (see Figure 2). The nodes on the surface of the sphere were given x, y, and z motional fixities to turn it into an infinitely hard sphere. The mesh density of the model was varied from 5 to 55 layers of elements in 10 meters to provide good computational speed at low ka and good fidelity at high ka where k is the wavenumber and a is the radius of the sphere. Computation time varied from 3 seconds for 5 element layers (210 nodes) to 3 hours for 55 element layers (15960 nodes).
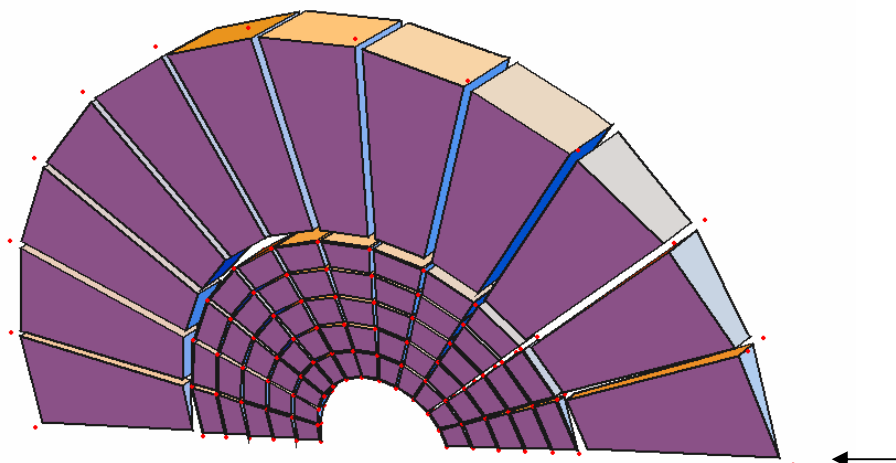

Figure 2. Low *ka* linear element model of rigid 1-meter sphere in water with 5 layers of elements to the far field radius.

A plane wave was impinged along the symmetry axis and plots of the backscattered target strength as a function of ka were developed. Also, polar plots of target strength as a function of angle at a given ka were generated.

2.2 Target Strength Versus Log *ka*

Target strength as a function of ka was generated from the rigid sphere model with 55 layers of elements to the far field radius of 10 meters. This led to a model with 15960 nodes.
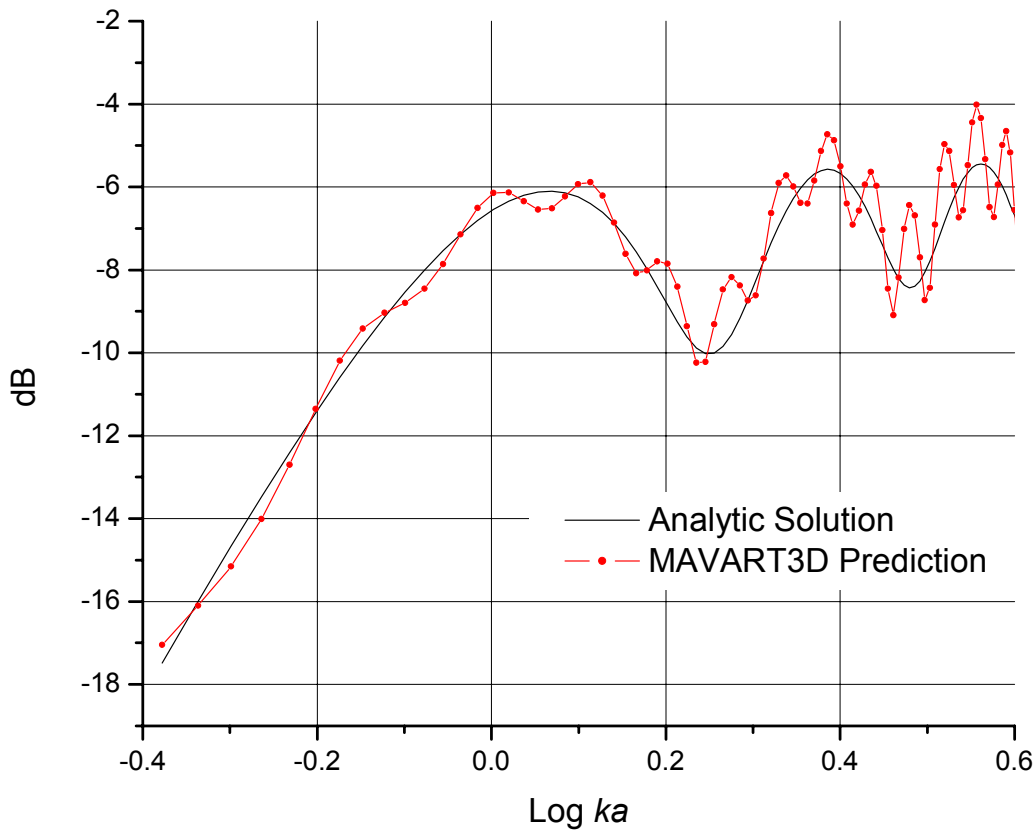
Figure 3. MAVART3D predicted target strength versus log *ka*.

Good agreement between MAVART3D predicted backscattered target strength and theory is seen, with a periodic error that grows to ± 1.2 dB of the analytic target strength at higher *ka* visible in Figure 3.

2.3 High and Low *ka* Results

In Figures 4 and 5, good agreement can be seen in the polar plots with maximum variation between the series solution and MAVART3D results of approximately 3 dB (dots represent MAVART3D results) at both low and high *ka*.

Target Strength of Sphere dB at ka = 0.20944

Target Strength dB at ka = 0.502655

Target Strength dB at ka = 1.0472

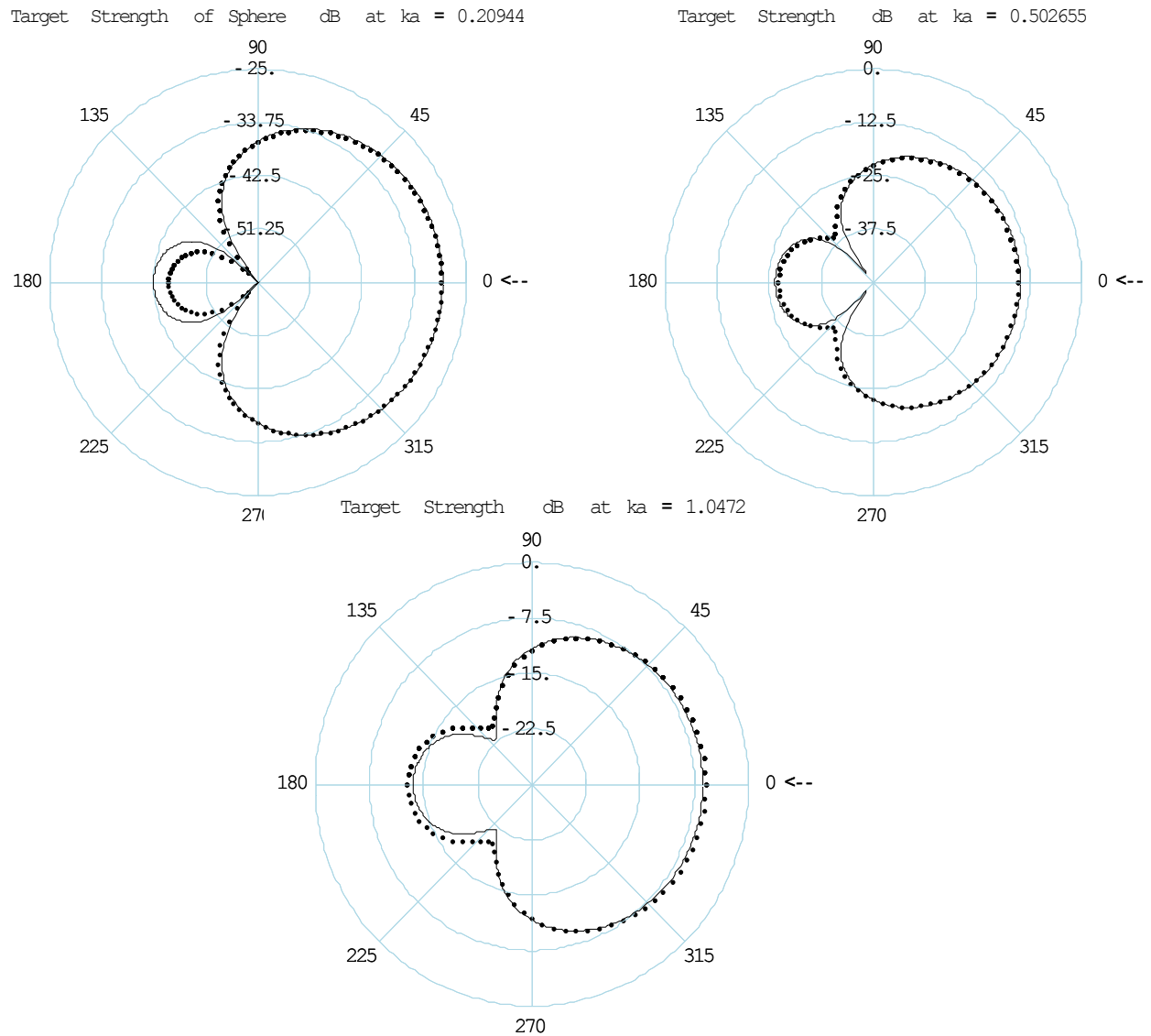**Figure 4. MAVART3D predicted target strengths (dots) versus series solution (solid line) at low *ka*.**

Target Strength   dB   at   ka = 2.0944

Target   Strength   of Sphere   dB   at   ka = 4.16114

Target   Strength   dB   at   ka = 8.37758

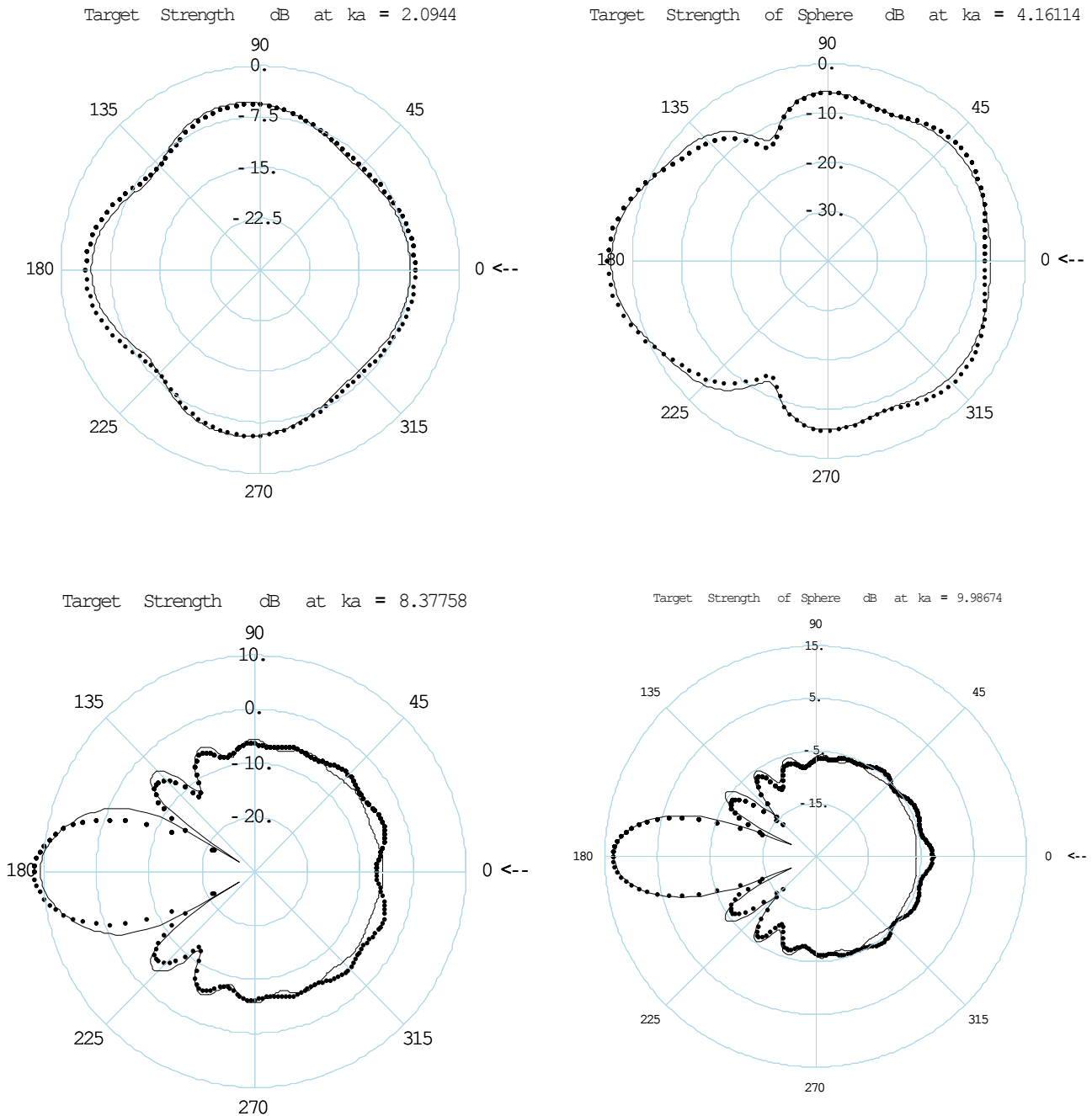Target   Strength   of Sphere   dB   at   ka = 9.98674

Figure 5. MAVART3D predicted target strengths (dots) versus analytic solution (solid line) at high *ka*.

3.0 RIGID CYLINDER SCATTERING

A steel-walled cylinder model has been used to test the FESTA and FE-VSC codes [7, 9]. Since the FE-VSC program uses the virtual source method and has been shown to be consistent with FESTA's fully meshed water approach with Bayliss-Turkel radiation boundary conditions, a comparison with MAVART3D's DAA method is of interest. Although MAVART3D is the only member of the MAVART family of codes that can do scattering, the radiating cylinder is a useful test of the consistency of all 3 MAVART codes by modelling radiation and thereby verifying the construction of the MAVART3D model. Since the 3 codes are written in different languages (Fortran 77, Fortran 90, and C++) and use different radiation algorithms, agreement between them would also provides reassuring evidence that a key component (DAA) of MAVART3D's scattering algorithm has been correctly implemented.

3.1 Cylinder Model

3.1.1 Model Verification Endcap Excitation

An axisymmetric model of a cylinder with a length of 60 cm overall, hemispherical endcaps of radius 9.08 cm, 1 mm wall thickness, density of 7600 kg/m^3, Poisson's ratio of 0.3 and Young's modulus of 200 GPa was generated in ModelMaker. The model was built using various element types including linear quadrilateral, quadratic quadrilateral and shell elements to compare their effectiveness and then run by all 3 MAVART codes (see Figure 6).
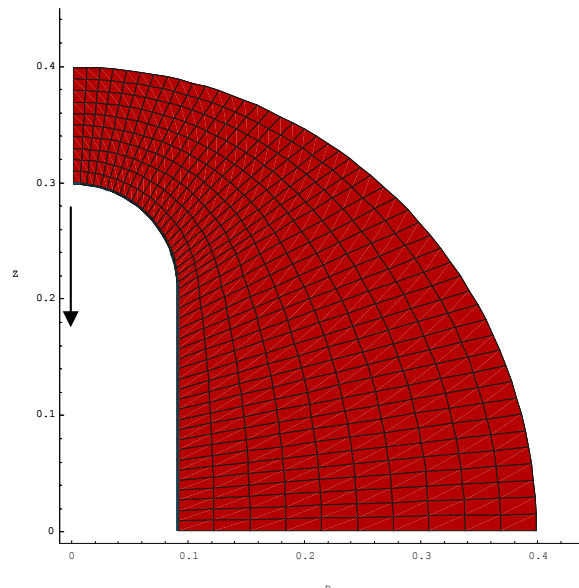

Figure 6. MAVART solid quadrilateral element axisymmetric model of hollow cylinder.

A 1 Newton force was applied axially to the tip of the hemispherical endcap and radiated power as a function of frequency was calculated using all three of the MAVART codes (see Figure 7). This confirms that the MAVART3D model was generated properly and that its radiation algorithm works.
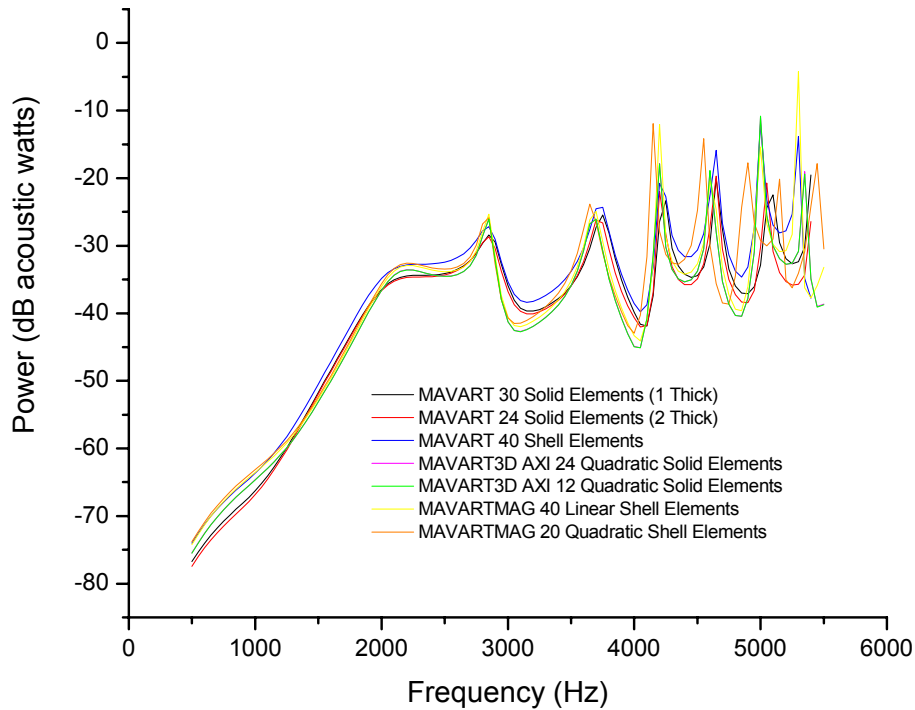

Figure 7. Acoustic power versus frequency with 1N force on top of cylinder endcap.

3.1.2 Cylinder Models

A MAVART3D n-fold symmetric model of the cylinder was built for scattering predictions (see Figure 8).
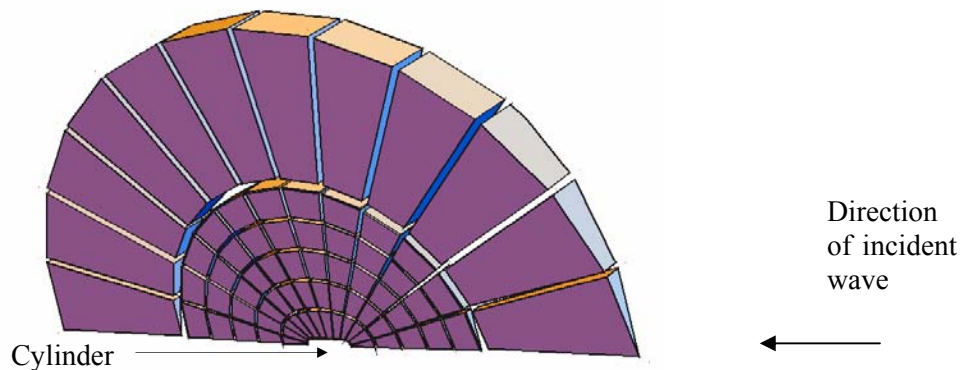

Figure 8. 5-element to far field radius n-fold symmetric model of cylinder

As in the rigid sphere case, the cylinders' surface nodes were given x, y and z motional fixities to make the cylinder rigid. The target strength of the rigid cylinder at 2500 Hz was estimated at a range of 4 meters and normalized to one meter. There were 35 layers of elements between the cylinder's surface and the 4 m far-field radius that resulted in 5 elements/λ.

The FE-VSC finite element model consisted of 200 shell elements and 200 virtual sources located 1.5 cm inside the shell elements. This model exploits azimuthal symmetry solving a two-dimensional problem for each Fourier order. In the case of end-on incidence, only the zeroth-order Fourier term is required.

3.2 Rigid Cylinder Target Strength Results

As seen in Figure 9, there is good agreement between MAVART3D and FE-VSC code target strength results, with at most 1.2 dB difference across all angles at 2500 Hz.
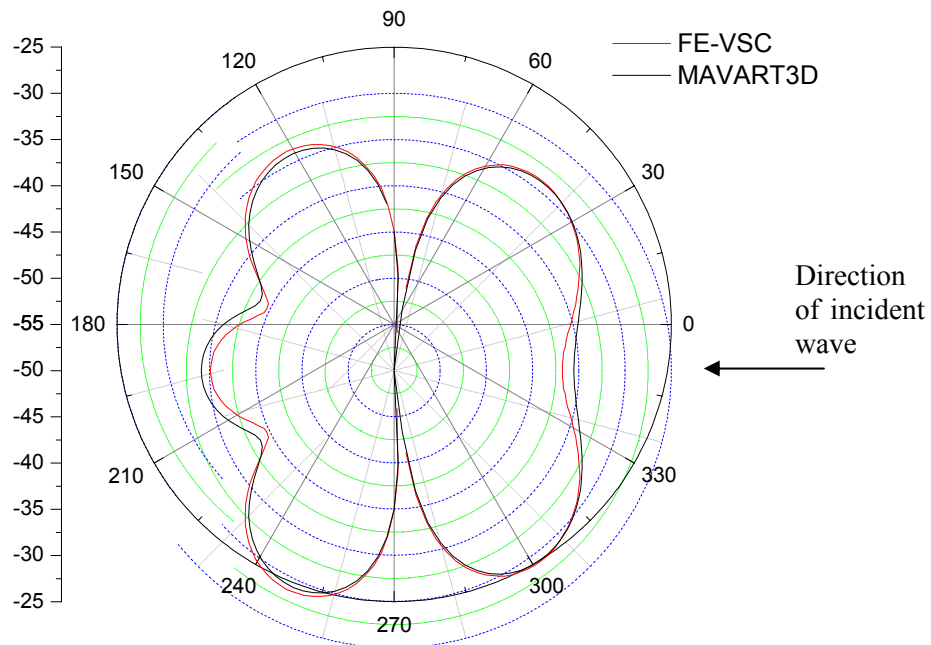

Figure 9. MAVART3D rigid cylinder 2500 Hz target strength compared to FE-VSC prediction.

4.0 ELASTIC CYLINDER SCATTERING

The MAVART3D model from the rigid cylinder case was used in the elastic cylinder case simply by removing all of the motional fixities except the symmetry plane translational and rotational fixities and the analysis was done at 2500 Hz. The FE-VSC geometry was the same as in the rigid case but now included the elastic response of the target (thin-shell theory).

The thick shell elements used in MAVART3D for the cylinder are based on Riessner-Mindlin theory [2]. In this shell element theory, the effect of shear deformation is taken into account and therefore an imaginary straight line perpendicular to the mid-surface plane does not necessarily remain perpendicular to that plane when undergoing deformation. An approximation for the actual shear stress is made with a constant equivalent shear component. A shear correction factor is added to the steel's stiffness matrix and is computed from the ratio of the shear area to the section area to account for the shear component. For a beam with a rectangular cross-section, the shear correction factor is 5/6 which was the value used in the cylinder model.

4.1 Elastic Cylinder Target Strength Results

MAVART3D's scattering results for the elastic cylinder with 1mm wall thickness are compared to the FE-VSC code predictions in Figure 10. The MAVART3D model had 42 layers of elements to the far field, 3956 total elements and 11308 nodes and required 33 hours of CPU time.

The general shapes of the polar plots of target strength are similar. However, the MAVART3D predicted forward scattered target strength is 0.7 dB less than the FE-VSC value and the MAVART3D backscattered target strength is about 3.4 dB less than the FE-VSC value. FE-VSC uses "thin" shell elements, which will be stiffer than MAVART3D's "thick" shells, and this could explain the differences in results.
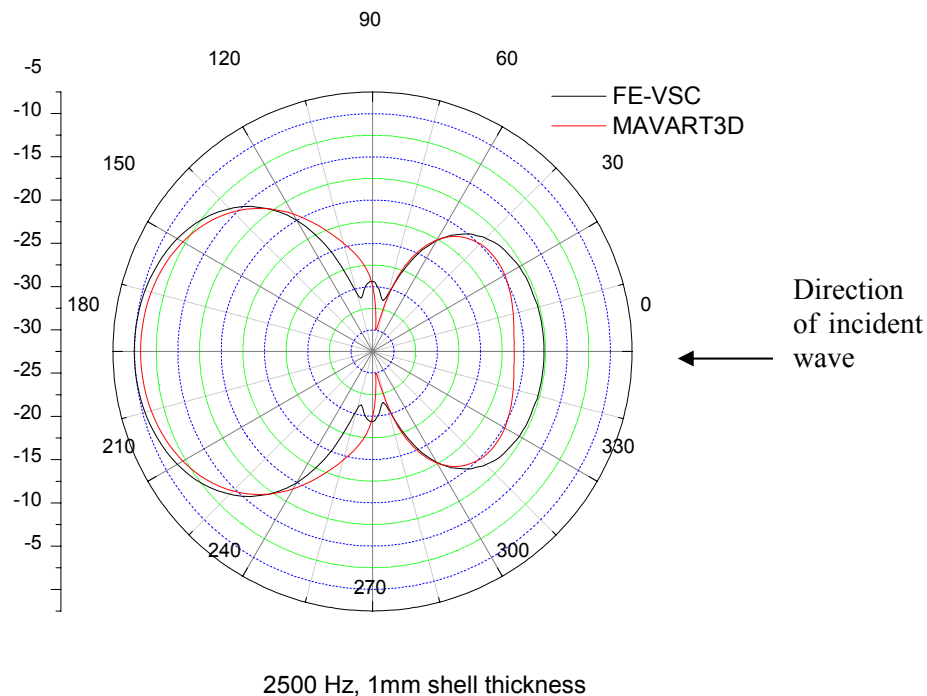

2500 Hz, 1mm shell thickness

Figure 10. MAVART3D and FE-VSC elastic cylinder 2500 Hz target strength results.

5.0 FUTURE WORK

As follow-on to this work, the MAVART3D code has recently been modified to permit its BEM elements to perform scattering calculations, which should enable work at higher *ka*. Tests similar to those described in this paper and simulation of parallel spheres as described by Burnett and Zampolli [9] will be carried out.

It would be useful to implement scattering with MAVART3D's axisymmetric elements since such models run very fast, and the endfire case for axisymmetric targets is of interest.

MAVART already has an infinite solid element developed for modelling axisymmetric seismic sensors on a simple sea bottom, but it would need a number of new elements developed to be able to deal with scattering from partially buried targets in poro-elastic media. Since there are both defence and commercial applications for this capability, this is on our wish list for future enhancements.

General efficiency improvements to both ModelMaker and MAVART3D would be useful to reduce computation time. These could include adding an option to invoke Mathematica's compiler early in the model generation process if symbolic model features are not required. We have not yet performed profiling on MAVART3D to check for routines in the program that might be optimised for speed and a further performance gain may be possible.

There is a requirement for a set of benchmark problems to be developed that would thoroughly exercise current scattering codes. The problems should also be designed to be experimentally verifiable, and a measurement program undertaken to collect scattering data from actual benchmark targets.

6.0 CONCLUSIONS

All three MAVART codes were used to model radiation from an elastic cylinder, and the results showed good agreement thereby confirming both the MAVART3D model was built correctly and the DAA portion of its scattering algorithm was functioning properly. MAVART3D's DAA algorithm was shown to be effective in modelling scattering of plane waves off an inelastic sphere in the range of *ka* from 0.2 to 10. MAVART3D's prediction of scattering off an inelastic cylinder agreed well with FE-VSC's prediction. MAVART3D's results for backscattering from an elastic cylinder disagreed by 3 dB with FE-VSC's results, but this difference may reflect the different shell elements in the two codes. This modelling effort revealed that the DAA method as implemented in MAVART3D requires excessive memory and computation time as frequency is raised. Further modelling of targets at high frequency will be carried out using the boundary element method in MAVART3D. This work suggests MAVART and ModelMaker, which were developed for transduction, also can be useful tools for solving acoustic engineering problems involving scattering.

7.0 ACKNOWLEDGEMENTS

We wish to thank Dr. Zdzislaw Wozniak, Coupled Systems Limited, Niagara Falls Ontario, author of MAVART3D and MAVARTMAG, for his assistance during this work.

8.0 REFERENCES

[1]   E.L. Skiba, G.W. MacMahon and Z. Wozniak, "Theoretical Manual for Program MAVART", DREA Contractor Report CR/95/415 (Limited Distribution), 1995.

[2]   Z. Wozniak, "Theoretical Manual for Program MAVART3D Version 4.3", DRDC Contractor Report Draft, 1999.

[3]   Z. Wozniak, "Theoretical Manual for Program MAVART-MAG Version 2.0", DRDC Contractor Report DRDC CR2004-026, 2004.

[4]   C. Purcell, "Building Finite Element Models with Mathematica", 1997 Mathematica Developers Conference, Wolfram Research Inc., Champagne, Illinois 1997, http://library.wolfram.com/infocenter/Conferences/229/

[5]   J.D. Pinter and C.J. Purcell, "MathOptimizer, An Advanced Modelling and Optimization System for Mathematica Users", Pinter Consulting Services, Halifax NS, 2002. http://is.dal.ca/~jdpinter/m_f_m.html

[6]   C. Purcell and R. Fleming, US Patent 6,584,039 Multi-Mode Pipe Projector, June 2003.

[7]   J. Fawcett, "Finite Element Modelling of Three-Dimensional Scattering from Azimuthally-Symmetric Elastic Shells", SACLANTCEN/SR-273, 1998.

[8]   P.M. Morse, "Vibration and Sound", 2nd Edition, McGraw Hill, 1948.

[9]   D.S. Burnett and M. Zampolli, "Development of a Finite-element, Steady-state 3-D Acoustic Code for Target Scattering", SACLANTCEN/SR-379, 2003.